Music - Tuning and Temperament

Terry Blackburn webSpace

Alternate Temperaments: Theory and Philosophy

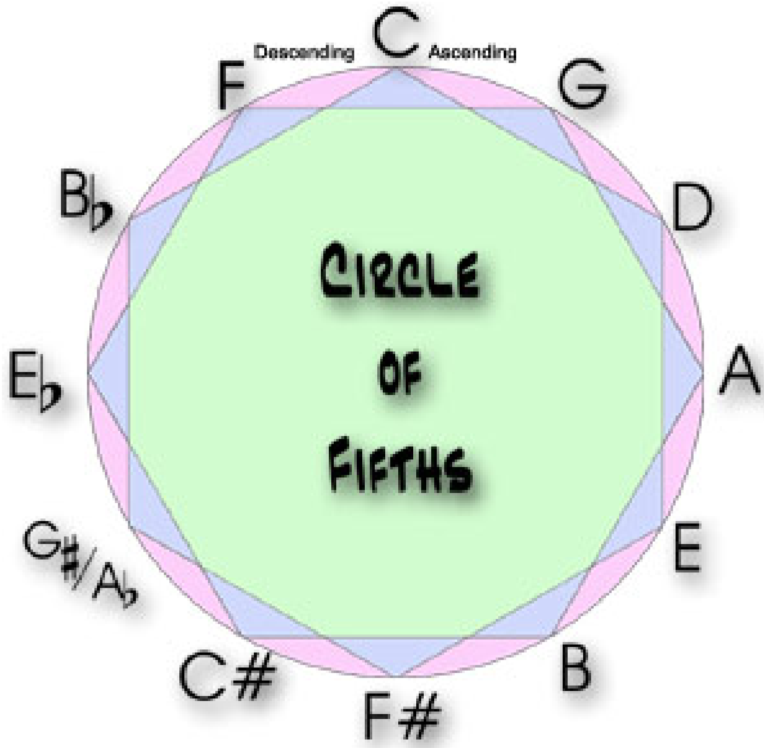
(The beginner's field guide to alternate temperaments)

INTRODUCTION:

Are you feeling just a bit temperamental? Well! Then this is the place for you! But if you're expecting to determine your sanguine to phlegmatic balance here, I'm sorry to disappoint you.

VERY, VERY BRIEF HISTORY [1]

Back in the old days (as long ago as yesterday in some circles, but nearly 200 years ago for the most part) instrument tuning was anything but a standard art. Clear back in the days of Pythagoras it was recognized that there are problems with creating a perfectly tuned scale.



Over the centuries there have been innumerable attempts to create tuning schemes that preserve the richness of perfectly tuned intervals while minimizing and distributing the errors that naturally occur when doing so.

CIRCLE OF FIFTHS AND HARMONIC SERIES

We'll talk about this 'error' and where it comes from in a minute. In order to continue, we must make some assumptions: First, you are familiar with the circle of fifths (c g d a e b f# c# g# d# a# (e#)f c). Second, you have a general working knowledge of frequency, harmonics (remember that long spring in physics class?) and the harmonic series.

Quickly, let's review the harmonic series. Suppose your fundamental frequency is 100hz (100 vibrations per second), the first harmonic is double that, or 200hz. The second harmonic is found at 300hz, third at 400hz, etc. Musically speaking, we know that when frequency doubles, pitch increases by exactly one octave. We have also discovered that the 2nd harmonic (300 hz) is exactly one octave and a pure [2] fifth higher than the fundamental (100hz).

The natural thing to want to do, then, is to tune the instrument so that each fifth is pure, all the way around the circle of fifths until you reach the note you started with. In doing so, you would expect a perfectly tuned scale. Ok, let's tune one. For the sake of simplicity, we're going to start our tuning at a frequency of **100**hz and we'll call it 'c' even though a real 'c' would be closer to 130-something. The first fifth would be tuned by ear by adjusting the pitch until a completely clear tone is produced with no beats. (Beats are that 'wah wah' sound that happens when your kids elementary school band is 'tuning up'.) If you put an electronic frequency analyzer on the string you tuned, you would find it vibrating 'g' at exactly **150**hz.

c	100	x 1.5 / 2 =
g	150	x 1.5 / 2 = 225
d	112.5	* divide by 2 again
a	168.75	
e	126.5625	*
b	189.8438	
f#	142.3828	*
c#	106.7871	*
g#	160.1807	
d#	120.1355	*
a#	180.2032	
(e#)f	135.1524	
C	202.7287	

frequencies marked with an *asterisk are divided by 2 one additional time to keep them within the desired octave

DO THE MATH...

Mathematically, that's the fundamental (100hz) times 3 (300hz for the second harmonic), divided by 2 to drop it back into the same octave as your starting pitch. This relationship is frequently expressed in terms of the ratio 3:2. If you had the luxury of a tuning hammer and a professional tuner to repair the damage, you could assault your own piano. I don't recommend it. Since we can't demonstrate this process auditorially, let's do the math for the rest of the scale. Tune the next fifth up - 150 * 3 = 450/2 = 225, still more than an octave above the starting pitch, so we'll drop it another octave to **112.5** 'd'. Moving on up... 112.5 * 3= 337.5 / 2 = **168.75** 'a' * 3 = 506.25 / 2 = 253.125 / 2 = **126.5625** 'e' * 3 = 379.6875 / 2= **189.84375** 'b' * 3 = 569.53125 / 4(see footnote 3) = **142.3828125** 'f#' * 3 = 427.1484375 / 4 = **106.787109375** 'c#' * 3 = 320.361328125 / 2 = **160.1806640625** 'g#' * 3 = 480.5419921875 / 4 = **120.1354980469** 'd#' * 3 = 360.4064941406 / 2 = **180.2032470703** 'a#' * 3 = 540.6097412109 / 4 = **135.1524353027** 'e#(f)' * 3 = 405.4573059082 / 2 = **202.7286529541** 'c'.

OOPS! Do you see the problem? Earlier, we predicted (guided by well understood and established laws of physics) the octave above c(100) would be c(200). When we ran the practical proof, using a circle of perfectly tuned fifths, we ended up at c(**202.7286529541),** wide by nearly 3 cycles**!** So, what gives? This is not a mathematical rounding error either. I purposely didn't round any of the answers so you could see that. If we had been tuning this on a real instrument, the results would be clear. We have a choice, either each fifth is perfectly tuned; a pure, rich sonority with octaves out of tune, or perfectly tuned octaves with the final fifth, f to c glaringly out of tune. It doesn't take much discussion or experience to realize that our ears will not tolerate mis-tuned octaves in any form!

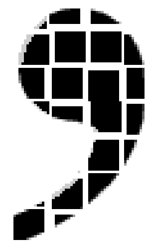
THE COMMA

This error, the difference between a perfectly tuned octave and the octave resulting from a tuned circle of fifths is known as the **COMMA** . For centuries now, musicians, mathematicians, theorists, tinkers, novices and experts have been trying to solve this conundrum.

Over the centuries numerous (or should that be innumerable?) schemes have been devised to accommodate this rather intrusive disruption of the musical fabric. The common consensus that has been in place for many years now is the concept of equal temperament [4]. However, getting to this point has not been an easy journey. Temperaments have been devised to maximize many different aspects of harmonic quality, all with compromises to others. Some maximize pure thirds (mean tone) while others emphasize pure fifths at the expense of the thirds (Kirnberger III).

Every temperament has its own unique 'character'. A piece of music may sound fine in one key, but really terrible in another. Transposing a piece to a new key can completely change its character. Careful attention must be paid to the selection of temperaments for authentic performances of historic keyboard music. A wrong choice could result in an unsatisfactory and historically inaccurate musical experience, or necessitate the complete re-tuning of the instrument before the concert can proceed.

WHY EQUAL TEMPERAMENT?



Equal temperament takes the tuning 'error' (comma, or the disparity between a pure octave and an octave generated by tuning perfect fifths), and spreads it equally between each step of a chromatic scale. The result is actually a scale of equally mis-tuned intervals, with no interval grossly out of tune, but none in perfect tune. Equal temperament has become the de facto standard for a two main reasons -

1. Convenience - It's quite troublesome to have to retune an instrument to a specific temperament that more adequately suits a particular piece of music. Many instruments are not capable of being alternately tuned (fretted string instruments in particular, and while the temperament of wind instruments cannot be altered, most allow sufficient pitch latitude to accommodate alternate temperaments).
2. Portability - every piece of important western music can be performed satisfactorily on an instrument tuned with equal temperament. Granted, some of the nuances may be missing for pieces that have their origins in another temperament, but pieces depending on equal temperament may be destroyed by the placement of various mis-tuned intervals.

WHY STUDY ALTERNATE TEMPERAMENTS?

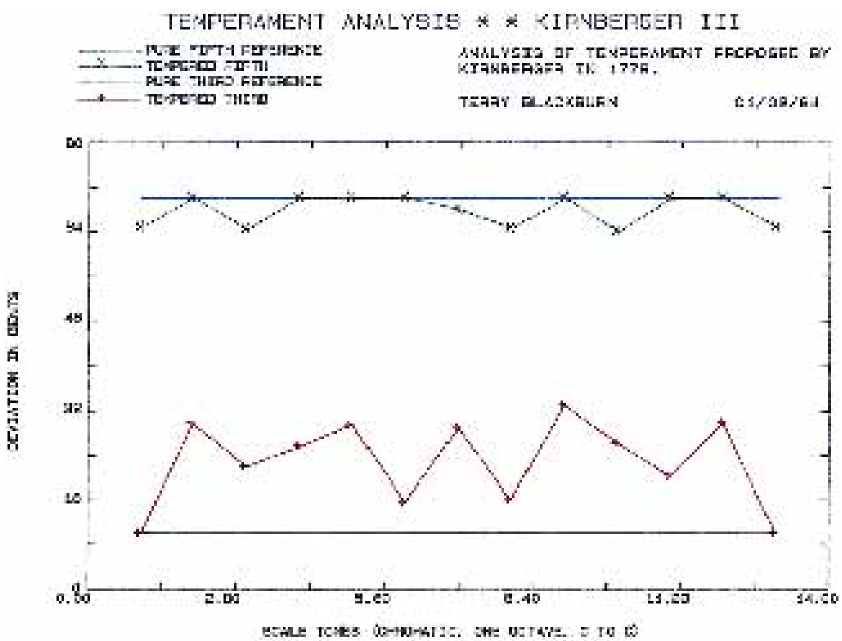
Many musicians listen and perform their entire careers without ever venturing into the realm of alternate temperament tuning. Most theoretical musicians must make at least one foray into this domain. Some venture in and and are so mesmerized by the complex possibilities and a perfection compulsion that they never emerge again. If you are just embarking on an adventure into alternate temperaments, a few words of advice would be in order.

- Be casual. Don't take it too seriously. Enjoy the stretch, hone your listening skills, lift your perceptions of pitch to a new level.
- Perfection is not possible. Mathematics proves this. Very many great minds have been

applied to the problem of creating the perfect scale. Even though your perspective is unique, you won't be the one to find it either. A solution does not exist.

- Take a historical perspective. The tension and release generated by music created to be played in an alternate temperament is a wonderful experience. Compare performances in equal temperament with a performance of the same piece in an alternate tuning. At first the differences are extremely subtle. As you become more accustomed to these subtleties, the results are astonishing and brilliant. It could change your entire perspective on how early music is to be performed. For example, many of the mysteries of baroque ornamentation become very clear.
- Let it go. Alternate temperaments aren't coming back into the main-stream any time soon (see two main reasons, above). Hold a wake, attend a grief and loss seminar, but get over it. Treasure the memories, but don't live in the past.

But, if you are a theoretical musician beginning your journey of destiny, you've come to the right place. You aren't the first to embark on this journey. Many have gone before. Many more will follow. Fortunately, the tools for the analysis of temperaments have improved dramatically in the past few years.



MY TEMPERAMENT ANALYSIS PROJECT

The information I have to share is part of a personal research project I undertook many years ago. In this project I graphically represented three aspects of a number of historic temperaments. First, I graphically plotted the pitch deviations of many alternate temperaments against a 'standard' equally tempered scale. Then, I compare the fifth and major thirds above each semi-tone of the chromatic scale against a 'pure' fifth and major third. In your analysis, remember that not all music is chromatic to the degree that every 'wildly' tuned interval will be realized during the course of a piece. Also remember, minor thirds have their own characteristics, and they are not part of this analysis. Fourths and sixths are simply inversions of fifths and thirds, and their characteristics are mirrors of their counterparts. The two types of charts created for this analysis are described in detail here .

Yes, this study is 'incomplete.' The reality is, I came, I heard, I learned, and I left [5]. My journey allowed me to learn and grow, and I have since let go of the driving urge to 'tune for perfection.' Should you wish to reproduce my results or expand on my analysis, I have included the raw data that fueled my study. A click on the title of each temperament will send you to the original graphs I produced in 1984.

Learning to tune temperaments for experiential analysis is an art in itself. While there is much of value to be learned, developing competent tuning skills can take months of intense practice. Modern technology has made it possible to create tools to circumvent some of these roadblocks. Many MIDI instruments allow fine pitch adjustments that can be explicitly assigned to each note. Let me warn you in advance - purists frown on any of these 'electronic' measures. Personally, I believe there is much to be gained and only very minor consequences from using these means.

On the side of empirical analysis, even the simplest of spreadsheet programs can now be enlisted to graph comparisons between pure and tempered thirds, fifths, or whatever. While these tools don't give you the auditory sense of the alternate temperament, they are extremely valuable in comparative analysis and selecting scalar characteristics to suit a particular study subject. Speaking of technological advancements, the graphs reproduced and displayed in the links below were produced by a custom computer program written in FORTRAN on a VAX 11/750. They represent a great deal of time and effort. Today, an afternoon with Microsoft Excel will produce far superior results.

Enjoy your studies. If you have comments, questions or suggestions, I'd love to hear them. E-mail Terry Blackburn

ANALYSIS OF HISTORIC TEMPERAMENTS

	Kirnberger II	Kirnberger III	Werckmeister III	Werckmeister IV	Werckmeister V	Werckmeister VI	Van Biezen	Bach (Klais)	Just (Barbour)
c	262.37	263.18	263.40	263.11	261.63	262.77	262.51	262.76	264.00
c#	276.40	277.26	277.50	275.93	276.56	276.83	277.18	276.87	275.00
d	295.16	294.25	294.33	294.66	294.33	292.77	294.00	294.30	297.00
d#	310.95	311.92	312.18	311.83	311.13	312.03	311.83	311.46	316.80
e	327.96	328.98	330.00	330.00	328.88	330.00	329.26	328.70	330.00
f	349.82	350.91	351.21	350.81	350.02	350.36	350.81	350.37	352.00
f#	368.95	370.10	369.99	369.58	369.99	370.53	369.58	369.18	371.25
g	393.55	393.55	393.77	392.88	392.44	393.39	392.88	393.70	396.00
g#	414.60	415.89	416.24	413.90	413.43	415.24	415.77	415.30	412.50
a	440.00	440.00	440.00	440.00	440.00	440.00	440.00	440.00	440.00
a#	466.43	467.88	468.27	469.86	466.69	468.05	467.75	467.18	475.20
b	491.93	493.47	495.00	492.77	493.33	495.00	492.76	492.26	495.00
c	524.73	526.36	526.81	526.21	523.25	525.54	525.03	525.53	528.00

	Pythagorean	van Zwolle	Meantone (-1/4)	Silbermann (-1/6)	Salinas (-1/3)	Zarlino (-2/7)	Rossi (-1/5)	Rossi (-2/9)	Rameau (syntoncic)
c	260.74	260.74	263.18	262.37	264.00	263.53	262.69	262.91	263.18
c#	278.44	274.69	275.00	276.14	273.86	274.51	275.68	275.38	276.71
d	293.33	293.33	294.25	293.94	294.55	294.38	294.06	294.14	294.25
d#	309.03	309.03	314.84	312.89	316.80	315.68	313.67	314.19	310.31
e	330.00	330.00	328.98	329.32	328.64	328.83	329.18	329.09	328.98
f	347.65	347.65	352.00	350.55	353.46	358.63	351.13	351.51	352.00
f#	371.25	366.25	367.81	368.95	366.67	367.32	368.49	368.19	368.95
g	391.11	391.11	393.55	392.73	394.36	393.90	393.06	393.28	393.55
g#	417.66	417.66	411.22	413.35	409.10	410.31	412.50	411.93	415.07
a	440.00	440.00	440.00	440.00	440.00	440.00	440.00	440.00	440.00
a#	463.54	463.54	470.79	486.36	473.24	471.84	469.33	469.98	467.39
b	495.00	495.00	491.93	492.25	490.92	491.50	492.55	492.27	491.93
c	521.48	521.48	526.36	524.73	528.00	527.06	525.38	525.82	526.36

Footnotes:

1 . For more depth into the history of tuning temperaments, see HISTORY OF TUNING AND TEMPERAMENT , an annotated outline by Howard Stoess

2 ·'Pure', in the tuning sense, as opposed to 'perfect' in the harmonic sense.

3 ·Dividing this frequency by 2 does not bring it down far enough to be in the same octave that we're trying to tune, which lies between 100hz and 200hz. Since this pitch needs to be brought down 2 octaves, we'll divide by 4 for simplicity, rather than by two twice.

4 . In comparison to all the art and effort that has gone into designing temperaments that minimize mis-tuning and preserve the beauty of perfectly tuned intervals, equal temperament is quite arbitrary. It is a purely mathematical solution to preserve the purity of the octave while spreading the comma equally throughout the scale. Each pitch of an equally tempered scale may be computed using the following formula - $P*2^{(x/12)}$ where P is your starting pitch ('a' 440hz is today's standard) and x is the scale degree (half steps, starting with zero for the first or unison pitch and proceeding in whole number increments to 12 for the octave). So, to calculate the pitch for 'c' using a(440) as our starting location, we calculate $440 * 2^{(3/12)}$ =523.2511306hz

5 . Veni, uh, Vidi,...no, um... oh, never mind...

Purchase temperament related merchandise, delivered to your doorstep through the magic of Cafe Press. No, this isn't a cheap attempt at crass commercialism as all proceeds benefit the Oregon Chorale. (The views expressed by the merchandise is in no way endorsed by the Chorale, its leadership, or its members.)

Return to Terry Blackburn's Music page

DISCLAIMER The opinions expressed on this page are my own. No warranty as to the accuracy or viability of the data presented here is either expressed or implied. While an honest effort has been made to ensure that facts are represented in an accurate manner, inadvertant errors may color the interpretation of the data. As with everything else you find on the internet, take what you read here with a grain of salt until proven through additional research and personal experience.